Published by Institute of Physics Publishing for SISSA
Received: *April 21, 2008*
Accepted: *June 10, 2008*
Published: *June 24, 2008*

Holographic QCD beyond the leading order

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Abstract: We consider a holographic QCD model for light mesons beyond the leading order in the context of 5-dim gauged linear sigma model on the interval in the AdS_5 space. We include two dimension-6 operators in addition to the canonical bulk kinetic terms, and study chiral dynamics of π, ρ, a_1 and some of their KK modes. As novel features of dim-6 operators, we get non-vanishing $\mathrm{Br}(a_1 \to \pi\gamma)$, the electromagnetic form factor and the charge radius of a charged pion, which improve the leading order results significantly and agree well with the experimental results.

Keywords: AdS-CFT Correspondence, QCD.

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1. Introduction

To understand the dynamics of low lying hadrons from underlying QCD has been a long standing problem in theoretical physics. In chiral Lagrangian approaches, it has been known for some time that the low energy dynamics of pions, vector mesons ρ and axial vector mesons a_1 are well described by the gauged linear sigma model (or its nonlinear version) with massive Yang-Mills gauge filds. The model Lagrangian up to dimension-6 operators is given by [1]

$$\begin{aligned}\mathcal{L}_{\text{MassiveYM}} = & \operatorname{Tr}\left[-\frac{1}{4}L_{\mu\nu}L^{\mu\nu} - \frac{1}{4}R_{\mu\nu}R^{\mu\nu} + \frac{1}{2}D_\mu\Phi D^\mu\Phi - \frac{1}{2}M_\Phi^2\Phi^\dagger\Phi\right] \\ & +\frac{1}{2}m_0^2 \operatorname{Tr}(L_\mu L^\mu + R_\mu R^\mu) \\ & +\operatorname{Tr}\left[+\zeta\left(L_{\mu\nu}D^\mu\Phi D^\nu\Phi^\dagger + R_{\mu\nu}D^\mu\Phi^\dagger D^\nu\Phi\right) + \kappa L_{\mu\nu}\Phi R^{\mu\nu}\Phi^\dagger\right] \\ & +\lambda_1\operatorname{Tr}(\Phi^\dagger\Phi)^2 + \lambda_2\left[\operatorname{Tr}(\Phi^\dagger\Phi)\right]^2 + (\lambda_3\det(\Phi) + H.c.)\end{aligned} \tag{1.1}$$

[1]We ignore the Wess-Zumino-Witten term in this work.

The role of the higher dimensional operators in light hadron dynamics, especially κ and ξ terms, were studied in the framework of the gauged linear sigma model in 4D [1, 2]. Although the above Lagrangian is quite successful in describing the $\pi - \rho - a_1$ system, it has a conceptual drawback in that we need to give gauge boson masses m_0^2 by hand. If we put $m_0^2 = 0$, global chiral symmetry becomes local symmetry, which is not a true symmetry of real QCD, and we end up with massless ρ and a_1, which is phenomenologically disastrous. Therefore we have to put $m_0^2 \neq 0$ and have to impose chiral symmetry only as a global symmetry. However, if chiral symmetry is only a global symmetry, then there is no compelling reason to introduce gauge covariant derivative, hence no reason for the minimal coupling between hadrons and (axial) vector mesons, and thus universality of the $P-V-V$ couplings. Since the universality seems to hold to a good approximation, it is tempting to implement global chiral symmetry to local symmetry. This have been remained a problem in chiral dynamics approach to the low lying hadrons.

Recently, there have been many interesting and successful attempts to understand hadron physics in the context of AdS/CFT correspondence [3]. The properties of hardrons and the hadron physics phenomenology are studied in various approaches [4−14] which are inspired by the AdS/CFT correspondence.

One may start with some stringy setup that may reproduce certain aspects of non-perturbative QCD. The most successful approach so far seems, arguably, the works by Sakai and Sugkimoto [5], and follow-up papers [15]. The model by Sakai and Sugimoto has nice features, but also some drawbacks. They show that chiral dynamics of π, ρ and a_1 system can be well reproduced by studying the N_f D8-branes in the background of N_c D4-branes. Also the Wess-Zumino-Witten (WZW) term is derived from the 5-dim Chern-Simon (CS) term. On the other hand, there are spurious SO(5) symmetry from S^5, which is not a true symmetry of real QCD. And it is not easy to accommodate nonzero quark masses, namely nonzero pion mass. Finally the pion and its radial excitation comes from different 5-dim fields, which is not easy to understand within the quark model. Despite numerous remarkable successes of Sakai-Sugimito model, there is an ample room for further improvement.

Independent of the stringy approach, a gravity dual model of the gauged linear sigma model was proposed to describe the chiral dynamics of light hadrons [6, 7]. This approach is often called the bottom-up approach, where one starts from QCD and then tries to construct its five-dimensional holographic dual model, AdS/QCD. Following the AdS/CFT correspondence, it is assumed that there are bulk fields that couple to the 4-dimensional QCD operators. For example, there are bulk gauge fields L_M and R_M that couple to the QCD operators $j_L \equiv \bar{q_L}\gamma^\mu q_L$ and $j_R \equiv \bar{q}_R\gamma^\mu q_R$, which are flavor currents.

Quality of the overall fit to the meson properties in the models of ref. [6, 7] is at the level of $\sim 30\%$, which is quite remarkable, considering the simplicity of the model. However it predicts $B(a_1 \to \pi\gamma) = 0$ and too small charge radius of a charged pion. The Lagrangian in ref. [6, 7] is the leading order one, since it contains only the bulk kinetic terms for the bulk gauge fields and scalar fields. In order to improve the predictions for $B(a_1 \to \pi\gamma)$ and the charge radius of a charged pion, we have to go beyond the leading order Lagrangian.

In this paper, we construct an AdS_5 dual model of the gauged linear sigma model with

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dimension-6 operators, motivated by the recently developed AdS/QCD model [6, 7]. To this end we incorporate higher dimensional operators, especially two dim-6 terms, into the AdS/QCD model.[2] In this work, we only consider the vector, axial-vector and pseudoscalar sectors, as a first step of our study. Interestingly enough, we find that the aforementioned problem of giving gauge boson masses m_0^2 is no longer present, since one can give masses of the vector and axial vector mesons, by projecting out the zero modes by choosing suitable boundary conditions. Degeneracy between the vector and the axial vector mesons will be lifted by the conventional Higgs mechanism. Still there remain physical pions.

Naively, these new operators will have nontrivial effects on the interaction vertex, as well as mass spectra and decay constants. We expect that they may contribute to the $\mathrm{Br}(a_1 \to \pi\gamma)$, which is zero in the original AdS/QCD model [6, 7]. We also study the phenomenology of $\rho \to \pi\pi$ and $a_1 \to \rho\pi$, the branching ratios and D/S wave amplitude ratio in the latter channel. By introducing photon as an external field, we study the pion electromagnetic form factor, and calculate the pion electromagnetic charge radius, which agrees with the experimental results in our numerical study.

This paper is organized as follows. In section 2, we define the Lagrangian of our model with two dim-6 operators in AdS_5. In section 3, we study the mass spectra and decay constants in vector, axial-vector and pseudoscalar sectors. We also present the interaction vertex and phenomenology of $a_1 \to \rho\pi$, $\rho \to \pi\pi$, $a_1 \to \pi\gamma$ channels, and calculate the pion charge radius. We derive the relevant chiral coefficients in section 4, and give our numerical results in section 5. The conclusions are drawn in section 6.

2. Gauged linear sigma model in the AdS$_5$ space

The Lagrangian of the holographic QCD model [6, 7] defined in a slice of AdS_5 is given by

$$\mathcal{L}_5^{\mathrm{dim}-4} = \sqrt{g} M_5 \ \mathrm{Tr}\ \Big[-\frac{1}{4} L_{MN} L^{MN} - \frac{1}{4} R_{MN} R^{MN} + \frac{1}{2} (D_M \Phi)^\dagger D^M \Phi - \frac{1}{2} M_\Phi^2 \Phi^\dagger \Phi \Big] , \tag{2.1}$$

where $M_\Phi^2 = -3/L^2$ from AdS/CFT correspondence [3], $D_M \Phi = \partial_M \Phi + i L_M \Phi - i \Phi R_M$, $L_M = L_M^a \tau^a/2$ with τ^a being the Pauli matrix, and $M, N = 0, 1, 2, 3, 5(\text{or } z\,)$. We define $\Phi = S e^{iP/v(z)}$ with $\langle S \rangle = v(z)$. Under $\mathrm{SU}(2)_V$, S and P transform as singlet and triplet, respectively. The AdS_5 space is characterized in the conformally flat metric with a warp factor $a(z) \equiv L/z$:

$$ds^2 = a^2(z)(dx^\mu dx_\mu - dz^2). \tag{2.2}$$

The scale L is the curvature of the 5-dimensional AdS space. In this model, the AdS_5 space is compactified such that $L_0 < z < L_1$, where $L_0 \to 0$ is an ultra-violet (UV) cutoff and L_1 is an infrared (IR) cutoff. Solving the equation of motion for S, we obtain [7]

$$\langle S \rangle \equiv v(z) = c_1 z + c_2 z^3 \tag{2.3}$$

[2]For a brief report on the present work, see Ref [16]

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with the integration constants $c_{1,2}$,

$$c_1 = \frac{M_q L_1^3 - \xi L_0^2}{L L_1 (L_1^2 - L_0^2)}, \qquad c_2 = \frac{\xi - M_q L_1}{L L_1 (L_1^2 - L_0^2)} \,. \tag{2.4}$$

Here we adopted the following boundary conditions

$$M_q = \frac{L}{L_0} v \bigg|_{L_0}, \qquad \xi = L v \bigg|_{L_1}, \tag{2.5}$$

where M_q is the current quark mass matrix, which breaks chiral symmetry explicitly, and ξ is related to $\langle \bar{q} q \rangle$, which breaks chiral symmetry spontaneously. The value of L_1 is fixed by the rho-meson mass: $1/L_1 \simeq 320$ MeV [6, 7]. There may be several ways to improve the model given above, though several observables obtained from the model are in agreement with experiments. One immediate extension of the model is to see corrections from various sources: trilinear or quartic interactions among the vector fields, 5D loop corrections, higher dimensional operators and back-reactions on the metric due to condensates [12]. In the present work, we consider corrections to the model from higher dimensional operators, though to be consistent we have to treat all those corrections at the same time. We note that a part of large N_c corrections through meson-loop contributions are discussed in ref. [18].

Now we introduce higher dimensional operators in the model Lagrangian in eq. (2.1). In principle, we can include infinite tower of higher dimensional operators, but for simplicity we consider only dimension-6 operators in the chiral limit. Note here that we have the following mass dimensions for a scalar field Φ and vector fields L_M and R_M:

$$\dim(\Phi) = \dim(L_M) = \dim(R_M) = 1 \,. \tag{2.6}$$

The Lagrangian with dimension-6 operators reads

$$\begin{aligned} \mathcal{L}_5^{\mathrm{dim-6}} = \sqrt{g} M_5 \; \mathrm{Tr}\bigg[- i \frac{\kappa}{M_5^2} \Big(L_{MN} D^M \Phi (D^N \Phi)^\dagger + R_{MN} (D^M \Phi)^\dagger D^N \Phi \Big) \\ + \frac{\zeta}{M_5^2} L_{MN} \Phi R^{MN} \Phi^\dagger \bigg] , \end{aligned} \tag{2.7}$$

where κ and ζ are constants that will be fixed later.

There are more dimension-6 operators, such as

$$\mathcal{L}_5^{\mathrm{dim-6}} = \sqrt{g} M_5 \; \mathrm{Tr}\Big[L_M^{\ N} L_N^{\ P} L_P^{\ M} + (L \leftrightarrow R) \Big] \tag{2.8}$$

However these terms are $O(p^6)$ after chiral symmetry breaking, whereas the κ and ζ terms are $O(p^4)$ after chiral symmetry breaking. Therefore we keep only those dimension-6 terms that reduce to $O(p^4)$ after chiral symmetry breaking. We note that the corrections to physical observables from the second dim-6 operator $\mathrm{Tr}[L_{MN} \Phi R^{MN} \Phi^\dagger]$ in eq. (2.7) and the operator in eq. (2.8) have been discussed in ref. [19].

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3. Vector, axial-vector and pseudoscalar sectors

3.1 Relevant parts of the lagrangian

In this section, we work in the chiral limit. Then $v(z)$ is proportional to $\mathbf{1}$, $v(z) \simeq \xi \frac{z^3}{L_1^3}\mathbf{1}$. The vector and axial gauge bosons are defined by

$$\begin{aligned} V_M &= \frac{1}{\sqrt{2}}(L_M + R_M) \\ A_M &= \frac{1}{\sqrt{2}}(L_M - R_M)\,. \end{aligned} \tag{3.1}$$

In order to cancel the mixing terms of V_μ, A_μ (μ as 4D Lorentz index, $0,1,2,3$) and V_z, A_z, P, we add gauge fixing terms

$$\begin{aligned} \mathcal{L}_{\rm GF}^V &= -\frac{M_5 a}{2\xi_V}\mathrm{Tr}\left[\partial_\mu V^\mu - \frac{\xi_V}{a}\left(\partial_5(aV_z) - \frac{2\zeta}{M_5^2}\partial_5(av^2 V_z)\right)\right]^2, \\ \mathcal{L}_{\rm GF}^A &= -\frac{M_5 a}{2\xi_A}\mathrm{Tr}\left[\partial_\mu A^\mu - \frac{\xi_A}{a}\left(\partial_5(aA_z) + \sqrt{2}a^3 vP\right.\right. \\ &\qquad \left.\left. + \frac{2\sqrt{2}\kappa}{M_5^2}\partial_5(a(\partial_5 v)P) + \frac{4\kappa}{M_5^2}av(\partial_5 v)A_z + \frac{2\zeta}{M_5^2}\partial_5(av^2 A_z)\right)\right]^2. \end{aligned} \tag{3.2}$$

In the unitary gauge, $\xi_{V,A} \to \infty$, we have the following relation between A_z and P,

$$\sqrt{2}a^3 vP + \partial_5(aA_5) + \frac{2\sqrt{2}\kappa}{M_5^2}\partial_5(a(\partial_5 v)P) + \frac{4\kappa}{M_5^2}av(\partial_5 v)A_z + \frac{2\zeta}{M_5^2}\partial_5(av^2 A_z) = 0, \tag{3.3}$$

which is identical to the leading order relation [7] when $\kappa = \zeta = 0$.

The quadratic terms for vector, axial-vector and pseudoscalar are given by, after integration by parts,

$$\begin{aligned} \mathcal{L}_V &= \frac{M_5}{2}a\mathrm{Tr}\left\{V_\mu\left(\partial^2 Z_v - a^{-1}\partial_5 a Z_v \partial_5\right)V^\mu\right\}, \\ \mathcal{L}_A &= \frac{M_5}{2}a\mathrm{Tr}\left\{A_\mu\left(\partial^2 Z_a - a^{-1}\partial_5 a Z_a \partial_5 + 2a^2 v^2 - \frac{8\kappa}{M_5^2}v(\partial_5 v)\partial_5\right)A^\mu\right\}, \\ \mathcal{L}_\pi &= \frac{M_5}{2}a\mathrm{Tr}\left\{(-2a^3 v^2)\left(A_z + \partial_5\frac{P}{\sqrt{2}v}\right)\right. \\ &\qquad \left. + a(\partial_\mu A_z)^2 + a^3(\partial_\mu P)^2 + \frac{4\sqrt{2}\kappa}{M_5^2}a(\partial_5 v)(\partial_\mu A_z)(\partial^\mu P) + \frac{2\zeta}{M_5^2}av^2(\partial_\mu A_z)^2\right\}. \end{aligned} \tag{3.4}$$

with $Z_v = 1 - \frac{2\zeta v^2}{M_5^2}$ and $Z_a = 1 + \frac{2\zeta v^2}{M_5^2}$. The boundary terms are

$$\begin{aligned} \mathcal{L}_{\rm boundary} &= M_5 a\mathrm{Tr}\left(V^\mu Z_v \partial_5 V_\mu + A^\mu Z_a \partial_5 A_\mu\right. \\ &\qquad \left.\left. - A_\mu \partial^\mu A_z - \frac{2\sqrt{2}\kappa}{M_5^2}(\partial_5 v)A_\mu \partial^\mu P - \frac{2\zeta}{M_5^2}v^2 A_\mu \partial^\mu A_z\right)\right|_{z=L_0}^{z=L_1}. \end{aligned} \tag{3.5}$$

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We choose the following boundary conditions to cancel the IR-boundary terms,

$$\partial_5 V_\mu\Big|_{z=L_1} = \partial_5 A_\mu\Big|_{z=L_1} = 0, \qquad V_5\Big|_{z=L_1} = A_5\Big|_{z=L_1} = 0 \tag{3.6}$$

$$A_z + \frac{2\sqrt{2}\kappa}{M_5^2}(\partial_5 v)P + \frac{2\zeta}{M_5^2}v^2 A_z\Big|_{z=L_1} = 0\,, \tag{3.7}$$

and the UV-boundary condition will be specified later.

We also calculated VAP, VPP and four-pion interaction vertices

$$\begin{aligned}
\mathcal{L}_{\rm VAP} = &\frac{\sqrt{2}i}{2}M_5 a\mathrm{Tr}\Big[A^\mu[\partial_5 V_\mu, A_z] - (\partial_5 A^\mu)[V_\mu, A_z] + \sqrt{2}a^2 v A^\mu[V_\mu, P]\Big]\\
&-\frac{2i\kappa}{M_5}a\mathrm{Tr}\Big[(\partial_5 v)(\partial_5 A^\mu)[V_\mu, P] - \sqrt{2}v(\partial_5 v)A^\mu[V_\mu, A_z] + \sqrt{2}v^2 A^\mu[\partial_5 V_\mu, A_z]\\
&\qquad\qquad +vA^\mu[\partial_5 V_\mu, \partial_5 P] + vA^\mu[V_{\mu\nu}, \partial^\nu P]\Big]\\
&-\frac{i\zeta}{M_5}av\mathrm{Tr}\Big[A^{\mu\nu}[V_{\mu\nu}, P] - 2(\partial_5 A^\mu)[\partial_5 V_\mu, P] + \sqrt{2}vA^\mu[\partial_5 V_\mu, A_z]\\
&\qquad\qquad +\sqrt{2}v(\partial_5 A^\mu)[V_\mu, A_z]\Big]\,,
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
\mathcal{L}_{V\pi\pi} = &\frac{i}{2}M_5 a\mathrm{Tr}\Big[V^\mu[A_z, \partial_\mu A_z] + a^2 V^\mu[P, \partial_\mu P]\Big]\\
&+\frac{\sqrt{2}i\kappa}{M_5}a\Big[\frac{1}{2}V^{\mu\nu}[\partial_\mu P, \partial_\nu P] + (\partial_5 V^\mu)[\partial_\mu P, \partial_5 P] - \sqrt{2}v(\partial_5 V^\mu)[A_z, \partial_\mu P]\\
&\qquad\qquad +\sqrt{2}(\partial_5 v)V^\mu[A_z, \partial_\mu P] - \sqrt{2}(\partial_5 v)V^\mu[\partial_\mu A_z, P]\Big]\\
&-\frac{\sqrt{2}i\zeta}{M_5}av\mathrm{Tr}\Big[-vV^\mu[A_z, \partial_\mu A_z] + \sqrt{2}(\partial_5 V^\mu)[P, \partial_\mu A_z]\Big]\,,
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\mathcal{L}_{\pi^4} = &-\frac{a^3 M_5}{12v^2}\mathrm{Tr}\Big[(\partial_\mu P)^2 P^2 - \Big((\partial_\mu P)P\Big)^2\Big]\\
&-\frac{a\kappa}{M_5}\mathrm{Tr}\Big[(\partial^\mu A_z)(\partial_\mu P)A_z P - \Big((\partial_\mu A_z)P\Big)^2\Big]\\
&+\frac{\sqrt{2}}{3}\frac{a(\partial_5 v)\kappa}{v^2 M_5}\mathrm{Tr}\Big[(\partial^\mu A_z)(\partial_\mu P)PP - (\partial^\mu A_z)P(\partial_\mu P)P\Big]\\
&-\frac{\sqrt{2}a\kappa}{vM_5}\mathrm{Tr}\Big[(\partial^\mu A_z)(\partial_\mu P)P(\partial_5 P) - (\partial^\mu A_z)(\partial_5 P)(\partial_\mu P)P\Big]\\
&-\frac{a\zeta}{M_5}\mathrm{Tr}\Big[(\partial_\mu A_z)^2 P^2 - \Big((\partial_\mu A_z)P\Big)^2\Big]\,.
\end{aligned} \tag{3.10}$$

3.2 Two-point correlation functions

We calculate the two-point correlation functions for vector and axial-vector with respect to the UV boundary external source fields v_μ and a_μ, which couple to the vector and axial-vector currents operators, respectively,

$$V_\mu|_{z=L_0} = v_\mu, \qquad A_\mu|_{z=L_0} = a_\mu. \tag{3.11}$$

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From the AdS/CFT correspondence, in order to calculate the current-current correlation function in the strongly coupled CFT side, we can do it in the weakly interacting AdS side instead. Then the effective Lagrangian in momentum space in term of the correlators is

$$\mathcal{L}_{\rm eff} = v_\mu \Pi_V^{\mu\nu}(p^2) v_\nu + a_\mu \Pi_A^{\mu\nu}(p^2) a_\nu \tag{3.12}$$

with $\Pi^{\mu\nu}_{V,A}(p^2) = (g^{\mu\nu} - p^\mu p^\nu/p^2)\,\Pi_{V,A}(p^2)$. We solve the equations of motion for the vector and axial-vector field derived from eq. (3.4) with the boundary conditions (3.6) and (3.11) and calculate the two-point correlation function

$$\Pi(p^2) = -M_5 L \frac{\partial_5 f(z)}{z f(z)}\bigg|_{z=L_0\to 0} \tag{3.13}$$

where $f(z)$ is the solution of differential equation. With dim-6 operators, we cannot calculate the two-point correlation function $\Pi(p^2)$ analytically, instead, we do it numerically.

For asymptotically large momentum $p^2 L_1^2 \gg 1$, we can expand the 2-point functions in powers of $1/p^2$, and get

$$\Pi_{V,A}(p^2) = p^2\left[\frac{M_5 L}{2}\ln p^2 L_0^2 + c_6^{V,A}\frac{1}{p^6}\right], \tag{3.14}$$

with

$$c_6^V = -\frac{192\zeta}{5M_5 L L_1^6}\xi^2, \qquad c_6^A = \left(\frac{16 M_5 L}{5L_1^6} + \frac{192\zeta}{5M_5 L L_1^6} + \frac{384\kappa}{5M_5 L L_1^6}\right)\xi^2, \tag{3.15}$$

which agree with the results in ref. [7] for $\kappa = \zeta = 0$. It is worthwhile to calculate the left-right correlator $\Pi_{LR} = \Pi_V - \Pi_A$, in the large momentum limit, we have

$$\Pi_{LR} = \frac{c_6}{p^4} + \ldots \tag{3.16}$$

with $c_6 = c_V - c_A$, where the experimental value of $c_6 = -4\pi\alpha_s\langle\bar{q}q\rangle^2 \simeq -1.3\times 10^{-3}\mathrm{GeV}^6$ is obtained from ref. [20]. We remark here that the vector correlator obtained in the present work and in the hard wall model [6, 7] has no $1/p^4$ compared to the results from operator product expansion (OPE) [21]. In the chiral limit, the coefficient of $1/p^4$ term is due to the gluon condensate [21]. In the hard wall model adopted in the present work, however, the metric is just a pure AdS with no gluon condensate included, and the model has no 5D bulk scalar field that couples to $\mathrm{tr}(G_{\mu\nu}G^{\mu\nu})$ at the boundary, where $G_{\mu\nu}$ is the gluon field strength tensor. Therefore, the vector and axial-vector correlators in the hard wall model do not contain $1/p^4$ term, as it should be. To have $1/p^4$ in eq. (3.14), we have to consider a deformed AdS background [22] due to the back-reaction of the gluon condensate.

In the large N_c limit, the above correlators can be written as the sum in terms of the resonance masses and decay constants,

$$\Pi_A(p^2) = p^2\sum_n \frac{f_{A_n}^2}{p^2 - M_{A_n}^2} + f_\pi^2 \tag{3.17}$$

$$\Pi_V(p^2) = p^2\sum_n \frac{f_{V_n}^2}{p^2 - M_{V_n}^2}. \tag{3.18}$$

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Then the vector and axial meson masses are determined as the poles of their corresponding correlators, and the decay constants are related with the residue,

$$f_{\rho,a_1}^2 = \lim_{p^2 \to m_{\rho,a_1}^2} (p^2 - m_{\rho,a_1}^2)\Pi_{V,A}(p^2)/p^2, \tag{3.19}$$

$$f_\pi^2 = \Pi_A(0). \tag{3.20}$$

4. Interactions and phenomenology

4.1 KK decompositions

Now we study hadronic observables such as decay widths and form factors using our model given in eq. (2.1) and eq. (2.7). Primarily we investigate how those dimension-6 operators in eq. (2.7) affect the results obtained with only interactions in eq. (2.1). To this end, we first Kaluza-Klein (KK) decompose the vector field as $V_\mu(x,z) = \frac{1}{\sqrt{M_5 L}} \sum_{n=1}^{\infty} \tilde{V}_\mu^{(n)}(x) f_V^{(n)}(z)$ and also for the axial-vector and pseudoscalar fields, where we omit the superscript index (n) when we consider the lowest KK mode. The first resonances of the vector, axial-vector and pseudoscalar fields are associated with ρ, a_1 and π respectively. The equations of motion for the vector, axial-vector and pseudoscalar fields are easily read off from eq. (3.4). To cancel the boundary terms, in addition to the IR boundary conditions given in eq. (3.6) and eq. (3.7), we impose the following UV boundary conditions

$$\begin{aligned} V_\mu\Big|_{z=L_0} &= 0, \\ A_\mu\Big|_{z=L_0} &= 0, \\ P\Big|_{z=L_0} &= 0\,. \end{aligned} \tag{4.1}$$

We obtain the wave function and mass spectra of various fields numerically with the normalization conditions:

$$\begin{aligned} \int_{L_0}^{L_1} dz \frac{a}{L} Z_v(z) f_V^{(m)}(z) f_V^{(n)}(z) &= \delta_{mn}, \\ \int_{L_0}^{L_1} dz \frac{a}{L} Z_a(z) f_A^{(m)}(z) f_A^{(n)}(z) &= \delta_{mn}, \\ \int_{L_0}^{L_1} dz \frac{a}{L}\left((f_{A_z}(z))^2 + a^2 (f_P(z))^2 + \frac{4\sqrt{2}\kappa}{M_5^2}(\partial_5 v) f_{A_z} f_P + \frac{2\zeta}{M_5^2} v^2 (f_{A_z})^2 \right) &= 1. \end{aligned} \tag{4.2}$$

4.2 $\rho \to \pi\pi$

The $\rho\pi\pi$ vertex can be expressed as

$$\mathcal{L}_{\rho\pi\pi} = \frac{i}{\sqrt{2}} g_{\rho\pi\pi} \mathrm{Tr}(\tilde{V}^\mu [\tilde{A}_5, \partial_\mu \tilde{A}_5]) + \frac{i}{\sqrt{2}} f_{\rho\pi\pi} \mathrm{Tr}(\tilde{V}^{\mu\nu} [\partial_\mu \tilde{A}_5, \partial_\nu \tilde{A}_5]) \tag{4.3}$$

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with the couplings

$$
\begin{aligned}
g_{\rho\pi\pi} &= \int_{L_0}^{L_1} dz \frac{a}{\sqrt{M_5 L^3}} \Bigg[f_V f_{A_z}^2 + a^2 f_V f_P^2 \\
&\quad + \frac{2\kappa}{M_5^2} \Big(-(\partial_5 f_V) f_P (\partial_5 f_P) - \sqrt{2} v (\partial_5 f_V) f_{A_z} f_P + 2\sqrt{2} (\partial_5 v) f_V f_{A_z} f_P \Big) \\
&\quad - \frac{2\zeta v}{M_5^2} \Big(-v f_V f_{A_z}^2 + \sqrt{2} (\partial_5 f_V) f_{A_z} f_P \Big) \Bigg] \qquad (4.4) \\
f_{\rho\pi\pi} &= \int_{L_0}^{L_1} dz \frac{\kappa}{\sqrt{M_5^5 L^3}} a f_V f_P^2 \qquad (4.5)
\end{aligned}
$$

We also calculate the decay width $\Gamma(\rho \to \pi\pi)$, which includes the non-minimal coupling $f_{\rho\pi\pi}$, even though its contribution is numerically small.

4.3 Electromagnetic form factor of a charged pion

Before we study the electromagnetic form factor of a charged pion, we introduce the photon as an external gauge field and rewrite the bulk vector field decomposition as

$$
V_\mu(x,z) = e\tilde{F}_\mu(x)\tau_3 + \frac{1}{\sqrt{M_5 L}} \sum_{n=1}^{\infty} \tilde{V}_\mu^{(n)}(x) f_V^{(n)}(z)\,, \qquad (4.6)
$$

with $\tau_3 = \sigma_3/\sqrt{2}$, where σ is the Pauli matrix, and e is identified with the physical electron charge at chiral symmetry breaking scale. To treat photon and ρ on the same footing, we introduce $f_F(z) = 1$ as the fifth dimension profile for photon. The advantage of our treatment of photon as external field, compared with the treatment of photon as the electromagnetic subgroup of $SU(3)_V$ [7], is that we don't need to worry about the KK excitations of the photon, as well as the mixing between photon KK excitations and ρ^0 KK excitations.

We consider the electromagnetic form factors of pions. In additional to the usual structure of contact $\gamma\pi\pi$ interaction $\mathrm{Tr}(F^\mu[\tilde{A}_5, \partial_\mu \tilde{A}_5])$, we also have non-minimal structure $\mathrm{Tr}(F^{\mu\nu}[\partial_\mu \tilde{A}_5, \partial_\nu \tilde{A}_5])$, which comes from the dim-6 κ term. And we also find $g_{\gamma\pi\pi} = e$ after comparing with the pion normalization condition, eq. (4.2). From the kinetic term (3.4) and the vector KK decomposition (4.6), we can derive the kinetic mixing of γ and ρ,

$$
\mathcal{L}_{\gamma\rho} = -\frac{1}{2} e g_{\gamma\rho} F^{\mu\nu} \tilde{V}_{\mu\nu}\,, \qquad (4.7)
$$

with

$$
g_{\gamma\rho} = \frac{M_5}{\sqrt{M_5 L}} \int_{L_0}^{L_1} dz a Z_v f_V(z)\,. \qquad (4.8)
$$

The electromagnetic form factor of pion can be calculated as

$$
F(q^2) = 1 - \frac{f_{\gamma\pi\pi}}{g_{\gamma\pi\pi}} q^2 - \frac{g_{\gamma\rho} q^2}{q^2 - m_\rho^2} g_{\rho\pi\pi}. \qquad (4.9)
$$

In small momentum limit, it can also be expressed as

$$
F(q^2) = 1 + \frac{1}{6} r_\pi^2 q^2 + \mathcal{O}(q^4)\,, \qquad (4.10)
$$

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with the pion charge radius r_π calculated as

$$r_\pi^2 = 6\left[-\frac{f_{\gamma\pi\pi}}{g_{\gamma\pi\pi}} + \frac{g_{\gamma\rho} g_{\rho\pi\pi}}{m_\rho^2} \right]. \tag{4.11}$$

Our vector meson dominance (VMD) is different from the usual VMDs as discussed in ref. [24], where we have an additional non-minimal $\gamma\pi\pi$ contact interaction, $\mathrm{Tr}(\tilde{F}^{\mu\nu}[\partial_\mu \tilde{A}_5, \partial_\nu \tilde{A}_5])$.

4.4 $a_1 \to \rho\pi$

We first consider the process $a_1 \to \rho\pi$. Applying the KK-decomposition to $\mathcal{L}_{\mathrm{VAP}}$ in eq. (3.8), we obtain

$$\begin{aligned} \mathcal{L}_{a_1\rho\pi} &= i g_{1a_1\rho\pi} \mathrm{Tr}(\tilde{A}^\mu [\tilde{V}_\mu, \tilde{A}_z]) + i g_{2a_1\rho\pi} \mathrm{Tr}(\tilde{A}^\mu [\tilde{V}_{\mu\nu}, \partial^\nu \tilde{A}_z]) \\ &\quad + i g_{3a_1\rho\pi} \mathrm{Tr}(\tilde{A}^{\mu\nu} [\tilde{V}_{\mu\nu}, \tilde{A}_z]) \end{aligned} \tag{4.12}$$

with the coefficients $g_{ia_1\rho\pi}$ $(i = 1, 2, 3)$

$$\begin{aligned} g_{1a_1\rho\pi} &= \int_{L_0}^{L_1} dz \frac{a}{\sqrt{M_5 L^3}} \Bigg[\frac{1}{\sqrt{2}} \Big(f_A (\partial_5 f_V) f_{A_z} - (\partial_5 f_A) f_V f_{A_z} + \sqrt{2} a^2 v f_A f_V f_P \Big) \\ &\quad - \frac{2\kappa}{M_5^2} \Big((\partial_5 v)(\partial_5 f_A) f_V f_P - \sqrt{2} v (\partial_5 v) f_A f_V f_{A_z} \\ &\qquad\qquad + \sqrt{2} a v^2 f_A (\partial_5 f_V) f_{A_z} + a v f_A (\partial_5 f_V)(\partial_5 f_P) \Big) \\ &\quad - \frac{\sqrt{2}\zeta}{M_5^2} a v \Big(v (\partial_5 f_A) f_V f_{A_z} + v f_A (\partial_5 f_V) f_{A_z} - \sqrt{2} (\partial_5 f_A)(\partial_5 f_V) f_P \Big) \Bigg] \end{aligned} \tag{4.13}$$

$$g_{2a_1\rho\pi} = -\int_{L_0}^{L_1} dz \frac{2\kappa}{\sqrt{M_5^5 L^3}} \Big[a v f_A f_V f_P \Big] \tag{4.14}$$

$$g_{3a_1\rho\pi} = -\int_{L_0}^{L_1} dz \frac{\sqrt{2}\zeta}{\sqrt{M_5^5 L^3}} \Big[a v^2 (\partial_5 f_A) f_V f_{A_z} \Big]. \tag{4.15}$$

With the interaction vertex above, it is straightforward to derive the amplitude of the process, which can be written as

$$\mathcal{A}(a_1 \to \rho\pi) = -i \epsilon^\mu(s_{a_1}) \epsilon^\nu(s_\rho) \Big[f_{a_1\rho\pi} g_{\mu\nu} + g_{a_1\rho\pi} p_{\pi\mu} p_{\pi\nu} \Big].$$

The S/D wave amplitudes are defined as in ref. [17]

$$\langle \rho(\vec{k} s_\rho) \pi(-\vec{k}) | H | a_1(0 s_{a_1}) \rangle = i f^S_{a_1\rho\pi} \delta_{s_\rho s_{a_1}} Y_{00}(\Omega_k) + i f^D_{a_1\rho\pi} \sum_{m_L} C(211; m_L s_\rho s_{a_1}) Y_{2m_L}(\Omega_k),$$

with

$$\begin{aligned} f^S_{a_1\rho\pi} &= \frac{\sqrt{4\pi}}{3m_\rho} \Big[(E_\rho + 2m_\rho) f_{a_1\rho\pi} - k^2 m_{a_1} g_{a_1\rho\pi} \Big] \\ f^D_{a_1\rho\pi} &= -\frac{\sqrt{8\pi}}{3m_\rho} \Big[(E_\rho - m_\rho) f_{a_1\rho\pi} - k^2 m_{a_1} g_{a_1\rho\pi} \Big]. \end{aligned} \tag{4.16}$$

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case	L_1	κ (10^{-6})	m_ρ	m_{a_1}	$\Gamma(\rho \to \pi\pi)$	$\Gamma(a_1 \to \pi\gamma)$	$\Gamma(a_1 \to \rho\pi)$
f_π	ξ	ζ (10^{-6})	f_ρ	f_{a_1}	$g_{\rho\pi\pi}$	r_π(fm)	D/S ratio
expected			775.8 ± 0.5	1230 ± 40	146.4 ± 1.5	0.640 ± 0.246	$250 \sim 600$
86.4 ± 9.7						0.672 ± 0.008	-0.108 ± 0.016
A	3.125	0.	[769.6]	[1253]	95.4	0.	295.5
85.0	4.0	0.	138	163	4.8	0.585	-0.055
B	2.836	-5.930	[775.8]	[1230]	[146.5]	0.088	165.3
71.9	2.56	-39.72	144	182	5.8	0.654	$[-0.094]$
C	3.102	-16.03	[775.8]	[1246]	[146.4]	0.042	409.8
[78.7]	4.010	0.09188	140	172	5.6	0.640	-0.027

Table 1: Various hadronic observables obtained in the present work. The unit of masses, decay constants and decay widths is MeV. The inputs for each case are shown in the brackets.

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And also the decay width of $a_1 \to \rho\pi$ is

$$\Gamma(a_1 \to \rho\pi) = \frac{p_c}{4\pi m_{a_1}} \left[\frac{2}{3} f_{a_1\rho\pi}^2 + \frac{1}{3} \left(\frac{E_\rho}{m_\rho} f_{a_1\rho\pi} + \frac{m_{a_1}}{m_\rho} p_c^2 g_{a_1\rho\pi} \right)^2 \right]. \tag{4.17}$$

4.5 $a_1 \to \pi\gamma$

In this subsection, we study the process $a_1 \to \pi\gamma$, With the help of vector KK decomposition eq. (4.6), we have similar results as $a_1 \to \rho\pi$. We have verified that the gauge non-invariant term of structure $\mathrm{Tr}(\tilde{A}^\mu [F_\mu, \tilde{A}_z])$ is cancelled out, when we impose the relation between A_z and P, e.g., eq. (3.3), the boundary condition eq. (3.7) and $\partial_5 f_F = 0$.

4.6 Numerical results

In this subsection, we present the numerical results of various hadronic obsevables and chiral coefficients discussed previously. We use χ^2 to fit the four parameters L_1, ξ, κ, ζ from m_ρ, m_{a_1}, D/S ratio, $\Gamma(\rho \to \pi\pi)$ in case B and m_ρ, m_{a_1}, $\Gamma(\rho \to \pi\pi)$, f_π in case C. Our results are summarized in table 1, 2, and figure 1. As a comparison, we also give Da Rold and Pomarol's results [7] in case A.

In both cases B and C, $\Gamma(a_1 \to \pi\gamma)$ is non-vanishing, but small (less than 100KeV), while $\Gamma(a_1 \to \rho\pi)$ is a little small in case B, but consistent with experimental measurement in case C. We have checked that the dominant contribution to $\Gamma(a_1 \to \rho\pi)$ comes from the leading order structure $\mathrm{Tr}(\tilde{A}^\mu [\tilde{V}_\mu, \tilde{A}_z])$. However, $\mathrm{Tr}(\tilde{A}^\mu [\tilde{F}_\mu, \tilde{A}_z])$ term is not gauge invariant and cancelled out for $a_1 \to \pi\gamma$ channel. Then only dim-6 κ and ζ terms contribute to the above process. This is different from usual 4D models with large $\Gamma(a_1 \to \pi\gamma)$, where the ratio between $\Gamma(a_1 \to \pi\gamma)$ and $\Gamma(a_1 \to \rho\pi)$ is roughly $e^2/g_{\rho\pi\pi}^2$, and only a single type of operator $\mathrm{Tr}(\tilde{A}^{\mu\nu}[\tilde{V}_{\mu\nu}, \pi])$ contributes to both channels [1].

The pion charge radius r_π agrees with the experiment in both case B and C. Pion decay constant f_π is a bit small in case B, while the D/S ratio of $a_1 \to \rho\pi$ is small in case C, compared with experiment. As in other 5D models, the KSRF relation $g_{\rho\pi\pi}^2/m_\rho^2 = c/f_\pi^2$ with $c = 1/2$ [23] is not satisfied very well. In both case B and C, c is roughly 0.3, which means the complete vector meson dominance of order $\mathcal{O}(p^2)$ four-pion interaction, with the higher ρ resonace and scalar exchange, and contact four-pion interaction contribution below $\sim 10\%$.

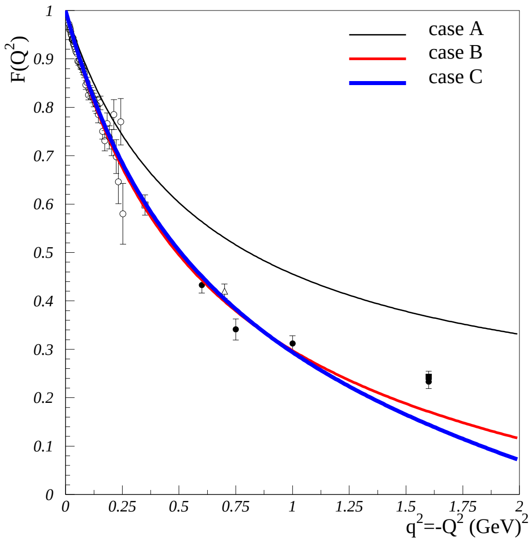

Figure 1: Pion form factor $F(Q^2)$ as a function of q^2. The white circles are data from CERN [25], square from DESY [26], triangle from DESY [27], black circle from Jlab [28], and black square from Jlab [29].

	case A	case B	case C	expected
c_6^V	0.	0.0008	0.0000	-0.0005
c_6^A	0.0014	0.0000	0.0006	0.0008
c_6	-0.0014	0.0008	-0.0006	-0.0013

Table 2: OPE coefficients c_6^V, c_6^A, and c_6 in unit GeV^6.

The pion form factor $F(q^2)$ as a function of q^2 is plotted in figure 1. We find the form factor has better behavior in case B and C for large value of momentum than that in case A.

The OPE coefficients c_6^V, c_6^A, and c_6 are presented in table 2. The individual coefficients c_6^V and c_6^A do not agree very well with the expected value in all three cases, while the coefficient of left-right correlator agrees with the expected value in case A. However, we note that the OPE is also sensitive to the deformation of the AdS metric [8]. Considering the OPE behavior, it may be worth remarking that, although the spirit of bottom-up AdS/QCD models has been to match the theory in the UV and then compare with the physical observables in the IR, it is not surprising that the best fit to data would arise from a model that disagrees with the precise UV behavior of QCD, where the model is not expected to be valid.

5. Chiral lagrangian for pseudoscalars up to $O(p^4)$

Before we discuss the $\mathcal{O}(p^4)$ chiral Lagrangian, we consider the vector field ρ effective

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case	L_1	L_2	L_3	L_9	L_{10}	$m_{\pi^+} - m_{\pi^0}$ (MeV)
exp	0.4 ± 0.3	1.4 ± 0.3	-3.5 ± 1.1	6.9 ± 0.7	-5.5 ± 0.7	4.6
A	0.43	0.86	-2.6	5.1	-5.5	3.4
B	0.32	0.65	-1.9	4.0	-5.0	1.5
C	0.46	0.93	-2.8	5.3	-5.1	2.9

Table 3: The chiral coefficients L_i in unit 10^{-3}.

Lagrangian [30],

$$\begin{aligned}\mathcal{L}_V &= -\frac{1}{4}\mathrm{Tr}[V^{\mu\nu}V_{\mu\nu}] + \frac{1}{2}m_\rho^2\mathrm{Tr}[V_\mu - \frac{i}{g}\Gamma_\mu]^2 \\ &\quad -\frac{1}{2\sqrt{2}}eg_{\gamma\rho}\mathrm{Tr}[V_{\mu\nu}f_+^{\mu\nu}] + \frac{i}{\sqrt{2}}f_{\rho\pi\pi}f_\pi^2\mathrm{Tr}[V_{\mu\nu}u^\mu u^\nu]\end{aligned} \tag{5.1}$$

with ρ transforming as gauge field of $\mathrm{SU}(2)_V$ and the notation of Γ_μ, $f_+^{\mu\nu}$, u^μ the same as in ref. [30]. The coefficients in the effective Lagrangian are determined by matching with our theory with dim-6 operators. The $\mathcal{O}(p^4)$ chiral Lagrangian for the pions is given in ref [31],

$$\begin{aligned}\mathcal{L}_4 &= L_1\mathrm{Tr}^2[D_\mu U^\dagger D^\mu U] + L_2\mathrm{Tr}[D_\mu U^\dagger D_\nu U]\mathrm{Tr}[D^\mu U^\dagger D^\nu U] + L_3\mathrm{Tr}[D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U] \\ &\quad + L_4\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\mathrm{Tr}[U^\dagger\chi + \chi^\dagger U] + L_5\mathrm{Tr}[D_\mu U^\dagger D^\mu U(U^\dagger\chi + \chi^\dagger U)] \\ &\quad + L_6\mathrm{Tr}^2[U^\dagger\chi + \chi^\dagger U] + L_7\mathrm{Tr}^2[U^\dagger\chi - \chi^\dagger U] + L_8\mathrm{Tr}[\chi^\dagger U\chi^\dagger U + U^\dagger\chi U^\dagger\chi] \\ &\quad - iL_9\mathrm{Tr}[F_R^{\mu\nu}D_\mu U D_\nu U^\dagger + F_L^{\mu\nu}D_\mu U^\dagger D_\nu U] + L_{10}\mathrm{Tr}[U^\dagger F_R^{\mu\nu}UF_{L\mu\nu}]\,.\end{aligned} \tag{5.2}$$

In the present, we do not discuss scalar and pseudoscalar resonances contribution to $L_{3,4,5,6,7,8}$, and only study the vector and axial resonances contribution to $L_{1,2,3,9,10}$. After Integrating out the vector rho meson, we obtain the following chiral coefficients,

$$\begin{aligned}L_1 &= \frac{f_\pi^4}{8m_\rho^4}g_{\rho\pi\pi}^2 - \frac{f_\pi^4}{4m_\rho^4}g_{\rho\pi\pi}f_{\rho\pi\pi}, \qquad L_2 = 2L_1, \qquad L_3 = -6L_1, \\ L_9 &= \frac{f_\pi^4}{m_\rho^4}g_{\rho\pi\pi}^2 + \frac{f_\pi^2}{2m_\rho^2}eg_{\rho\pi\pi}f_{\rho\pi\pi} - \frac{2f_\pi^4}{m_\rho^2}g_{\gamma\rho}g_{\rho\pi\pi}.\end{aligned} \tag{5.3}$$

L_{10} can be calculated from the two-point correlators of vector and axial, $\Pi_{V,A}$,

$$L_{10} = \frac{1}{4}[\Pi_A'(0) - \Pi_V'(0)], \tag{5.4}$$

where the derivative is over p^2.

We also calculate the electromagnetic mass difference of the pions from the operator of $\mathrm{Tr}[Q_R U Q_L U^\dagger]$,

$$m_{\pi^+} - m_{\pi^0} \simeq \frac{3\alpha_{\rm em}}{8\pi m_\pi f_\pi^2}\int_0^\infty dp^2(\Pi_A - \Pi_V). \tag{5.5}$$

The chiral coefficients of relevance and electromagnetic pion mass difference are given in table 3. Compared with Da Rold and Pomarol's case, the results do not significantly change much in our two cases.

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6. Conclusions

In this paper, we considered holographic QCD beyond the leading order, by including two dim-6 dimension operators that go beyond the usual quadratic kinetic terms for the bulk gauge field L_M and R_M, and scalar field Φ [6, 7]. We have studied the mass spectra, decay constants of vector, axial and pseudoscalar sectors, and phenomenology of $a_1 \to \rho\pi$, $\rho \to \pi\pi$ and $a_1 \to \pi\gamma$ channels. In our work, we could achieve a non-vanishing branching ratio for $a_1 \to \pi\gamma$, which is a new feature compared with the usual holographic QCD in the leading order. We also calculated the electromagnetic form factor of a charged pion, (including the charge radius of a pion) which agrees with the experimental results up to $q^2 \simeq 2\,\mathrm{GeV}^2$. The numerical results are summarized in table 1, and compared with the leading order results obtained by Da Rold and Pomarol [7] denoted as the case A. We could achieve significant improvements in overall phenomenology of the $\pi - \rho - a_1$ system by including the κ and ζ terms.

Let us remind ourselves that most studies based on the AdS/QCD approach are just the leading order calculations, starting from the bulk Lagrangian which is quadratic in the bulk gauge fields. Including the next-to-leading order corrections would be the next step to follow, and our present work makes such an attempt by considering dim-6 operators that reduce to the $O(p^4)$ operators after chiral symmetry breaking. Considering the improvement of overall phenomenology obtained in this work, it would be clearly desirable to have more systematic study of subleading corrections within AdS/QCD.

Acknowledgments

This work was supported in part by KOSEF SRC program through CHEP at Kyungpook National University.

References

[1] U.G. Meissner, *Low-energy hadron physics from effective chiral lagrangians with vector mesons*, *Phys. Rept.* **161** (1988) 213.

[2] P. Ko and S. Rudaz, *Phenomenology of scalar and vector mesons in the linear σ-model*, *Phys. Rev.* **D 50** (1994) 6877.

[3] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik and I. Kirsch, *Chiral symmetry breaking and pions in non-supersymmetric gauge/gravity duals*, *Phys. Rev.* **D 69** (2004) 066007 [hep-th/0306018];
M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Towards a holographic dual of large-N_c QCD*, *JHEP* **05** (2004) 041 [hep-th/0311270].

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[5] T. Sakai and S. Sugimoto, *Low energy hadron physics in holographic QCD*, *Prog. Theor. Phys.* **113** (2005) 843 [hep-th/0412141]; *More on a holographic dual of QCD*, *Prog. Theor. Phys.* **114** (2006) 1083 [hep-th/0507073].

[6] J. Erlich, E. Katz, D.T. Son and M.A. Stephanov, *QCD and a holographic model of hadrons*, *Phys. Rev. Lett.* **95** (2005) 261602 [hep-ph/0501128].

[7] L. Da Rold and A. Pomarol, *Chiral symmetry breaking from five dimensional spaces*, *Nucl. Phys.* **B 721** (2005) 79 [hep-ph/0501218].

[8] J. Hirn, N. Rius and V. Sanz, *Geometric approach to condensates in holographic QCD*, *Phys. Rev.* **D 73** (2006) 085005 [hep-ph/0512240].

[9] S.J. Brodsky and G.F. de Teramond, *Hadronic spectra and light-front wavefunctions in holographic QCD*, *Phys. Rev. Lett.* **96** (2006) 201601 [hep-ph/0602252].

[10] N. Evans and A. Tedder, *Perfecting the ultra-violet of holographic descriptions of QCD*, *Phys. Lett.* **B 642** (2006) 546 [hep-ph/0609112].

[11] D.K. Hong, T. Inami and H.-U. Yee, *Baryons in AdS/QCD*, *Phys. Lett.* **B 646** (2007) 165 [hep-ph/0609270].

[12] J.P. Shock, F. Wu, Y.-L. Wu and Z.-F. Xie, *AdS/QCD phenomenological models from a back-reacted geometry*, *JHEP* **03** (2007) 064 [hep-ph/0611227].

[13] T. Hambye, B. Hassanain, J. March-Russell and M. Schvellinger, *Four-point functions and kaon decays in a minimal AdS/QCD model*, *Phys. Rev.* **D 76** (2007) 125017 [hep-ph/0612010].

[14] K. Ghoroku, N. Maru, M. Tachibana and M. Yahiro, *Holographic model for hadrons in deformed AdS_5 background*, *Phys. Lett.* **B 633** (2006) 602 [hep-ph/0510334];
G. Panico and A. Wulzer, *Effective action and holography in 5D gauge theories*, *JHEP* **05** (2007) 060 [hep-th/0703287];
E. Katz and M.D. Schwartz, *An eta primer: solving the* U(1) *problem with AdS/QCD*, *JHEP* **08** (2007) 077 [arXiv:0705.0534];
U. Gürsoy and E. Kiritsis, *Exploring improved holographic theories for QCD: part I*, *JHEP* **02** (2008) 032 [arXiv:0707.1324];
U. Gürsoy, E. Kiritsis and F. Nitti, *Exploring improved holographic theories for QCD: part II*, *JHEP* **02** (2008) 019 [arXiv:0707.1349];
H.J. Kwee and R.F. Lebed, *Pion form factors in holographic QCD*, *JHEP* **01** (2008) 027 [arXiv:0708.4054].

[15] K. Nawa, H. Suganuma and T. Kojo, *Baryons in holographic QCD*, *Phys. Rev.* **D 75** (2007) 086003 [hep-th/0612187];
D.K. Hong, M. Rho, H.-U. Yee and P. Yi, *Chiral dynamics of baryons from string theory*, *Phys. Rev.* **D 76** (2007) 061901 [hep-th/0701276]; *Dynamics of baryons from string theory and vector dominance*, *JHEP* **09** (2007) 063 [arXiv:0705.2632];
H. Hata, T. Sakai, S. Sugimoto and S. Yamato, *Baryons from instantons in holographic QCD*, hep-th/0701280.

[16] X.H. Wu, *Gauged linear sigma model in AdS5 space and hadron physics*, talk given at *International Workshop on the Origin of Mass and Strong Coupling Gauge Theories (SCGT 06)*, Nagoya, Japan, 21–24 November (2006), published in *Nagoya* 2006*, the origin of mass and strong coupling gauge theories.*

JHEP06(2008)094

[17] N. Isgur, C. Morningstar and C. Reader, *The A1 in τ decay*, *Phys. Rev.* **D 39** (1989) 1357.

[18] M. Harada, S. Matsuzaki and K. Yamawaki, *Implications of holographic QCD in ChPT with hidden local symmetry*, *Phys. Rev.* **D 74** (2006) 076004 [hep-ph/0603248].

[19] H.R. Grigoryan, *Dimension six correction to the vector sector of AdS/QCD model*, *Phys. Lett.* **B 662** (2008) 158 [arXiv:0709.0939].

[20] M. Jamin, *Flavour-symmetry breaking of the quark condensate and chiral corrections to the Gell-Mann-Oakes-Renner relation*, *Phys. Lett.* **B 538** (2002) 71 [hep-ph/0201174].

[21] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *QCD and resonance physics. Sum rules*, *Nucl. Phys.* **B 147** (1979) 385.

[22] A. Kehagias and K. Sfetsos, *On running couplings in gauge theories from type-IIB supergravity*, *Phys. Lett.* **B 454** (1999) 270 [hep-th/9902125];
S.S. Gubser, *Dilaton-driven confinement*, hep-th/9902155;
C. Csaki, J. Erlich, C. Grojean, and T.J. Hollowood, *General properties of the selftuning domain wall approach to the cosmological constant problem*, *Nucl. Phys.* **B 584** (200) 359 [hep-th/0004133];
C. Csáki and M. Reece, *Toward a systematic holographic QCD: a braneless approach*, *JHEP* **05** (2007) 062 [hep-ph/0608266].

[23] K. Kawarabayashi and M. Suzuki, *Partially conserved axial vector current and the decays of vector mesons*, *Phys. Rev. Lett.* **16** (1966) 255;
Riazuddin and Fayyazuddin, *Algebra of current components and decay widths of ρ and K^* mesons*, *Phys. Rev.* **147** (1966) 1071.

[24] H.B. O'Connell, B.C. Pearce, A.W. Thomas and A.G. Williams, *$\rho - \omega$ mixing, vector meson dominance and the pion form-factor*, *Prog. Part. Nucl. Phys.* **39** (1997) 201 [hep-ph/9501251].

[25] S.R. Amendolia et al., *Measurement of the pion form-factor in the timelike region for Q^2 values between .1 GeV/c^2 and .18 GeV/c^2*, *Phys. Lett.* **B 138** (1984) 454;
NA7 collaboration, S.R. Amendolia et al., *A measurement of the space-like pion electromagnetic form-factor*, *Nucl. Phys.* **B 277** (1986) 168.

[26] P. Brauel et al., *Separation of Σ_L and Σ_U in π^+ electroproduction above the resonance region*, *Phys. Lett.* **B 69** (1977) 253; *Electroproduction of $\pi^+ n, \pi^- p$ and $K^+\Lambda, K^+\Sigma_0$ final states above the resonance region*, *Z. Physik* **C 3** (1979) 101.

[27] H. Ackermann et al., *Determination of the longitudinal and the transverse part in π^+ electroproduction*, *Nucl. Phys.* **B 137** (1978) 294.

[28] Jefferson Lab F(pi) collaboration, V. Tadevosyan et al., *Determination of the pion charge form factor for $Q^2 = 0.60$–$1.60 GeV^2$*, *Phys. Rev.* **C 75** (2007) 055205 [nucl-ex/0607007].

[29] Jefferson Lab F(pi)-2 collaboration, T. Horn et al., *Determination of the charged pion form factor at $Q^2 = 1.60$ and 2.45 $(GeV/c)^2$*, *Phys. Rev. Lett.* **97** (2006) 192001 [nucl-ex/0607005].

[30] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Chiral lagrangians for massive spin 1 fields*, *Phys. Lett.* **B 223** (1989) 425.

[31] G. Ecker, J. Gasser, A. Pich and E. de Rafael, *The role of resonances in chiral perturbation theory*, *Nucl. Phys.* **B 321** (1989) 311.

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